7.5 Work

- Find the work done by a constant force.
- Find the work done by a variable force.

Work Done by a Constant Force

The concept of work is important to scientists and engineers for determining the energy needed to perform various jobs. For instance, it is useful to know the amount of work done when a crane lifts a steel girder, when a spring is compressed, when a rocket is propelled into the air, or when a truck pulls a load along a highway.

In general, **work** is done by a force when it moves an object. If the force applied to the object is *constant*, then the definition of work is as follows.

Definition of Work Done by a Constant Force

If an object is moved a distance D in the direction of an applied constant force F, then the **work** W done by the force is defined as W = FD.

There are four fundamental types of forces—gravitational, electromagnetic, strong nuclear, and weak nuclear. A **force** can be thought of as a *push* or a *pull*; a force changes the state of rest or state of motion of a body. For gravitational forces on Earth, it is common to use units of measure corresponding to the weight of an object.

EXAMPLE 1

Lifting an Object

Determine the work done in lifting a 50-pound object 4 feet.

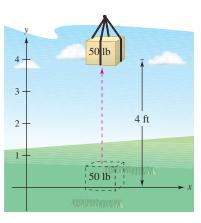
Solution The magnitude of the required force F is the weight of the object, as shown in Figure 7.46. So, the work done in lifting the object 4 feet is

W = FD	Work = $(force)(distance)$
= 50(4)	Force $= 50$ pounds, distance $= 4$ feet
= 200 foot-pounds.	

In the U.S. measurement system, work is typically expressed in foot-pounds (ft-lb), inch-pounds, or foot-tons. In the International System of Units (SI), the basic unit of force is the **newton**—the force required to produce an acceleration of 1 meter per second per second on a mass of 1 kilogram. In this system, work is typically expressed in newton-meters, also called joules. In another system, the centimeter-gram-second (C-G-S) system, the basic unit of force is the **dyne**—the force required to produce an acceleration of 1 centimeter per second per second on a mass of 1 gram. In this system, work is typically expressed in dyne-centimeters (ergs) or newton-meters (joules).

Exploration

How Much Work? In Example 1, 200 foot-pounds of work was needed to lift the 50-pound object 4 feet vertically off the ground. After lifting the object, you carry it a horizontal distance of 4 feet. Would this require an additional 200 foot-pounds of work? Explain your reasoning.



The work done in lifting a 50-pound object 4 feet is 200 foot-pounds. **Figure 7.46**

Work Done by a Variable Force

In Example 1, the force involved was *constant*. When a *variable* force is applied to an object, calculus is needed to determine the work done, because the amount of force changes as the object changes position. For instance, the force required to compress a spring increases as the spring is compressed.

Consider an object that is moved along a straight line from x = a to x = b by a continuously varying force F(x). Let Δ be a partition that divides the interval [a, b] into n subintervals determined by

$$a = x_0 < x_1 < x_2 < \cdots < x_n = b$$

and let $\Delta x_i = x_i - x_{i-1}$. For each *i*, choose c_i such that

$$x_{i-1} \leq c_i \leq x_i$$

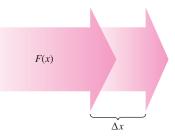
Then at c_i , the force is $F(c_i)$. Because F is continuous, you can approximate the work done in moving the object through the *i*th subinterval by the increment

$$\Delta W_i = F(c_i) \,\Delta x$$

as shown in Figure 7.47. So, the total work done as the object moves from a to b is approximated by

$$W \approx \sum_{i=1}^{n} \Delta W_i$$
$$= \sum_{i=1}^{n} F(c_i) \, \Delta x_i.$$

This approximation appears to become better and better as $\|\Delta\| \rightarrow 0 \ (n \rightarrow \infty)$. So, the work done is



The amount of force changes as an object changes position (Δx) . Figure 7.47



EMILIE DE BRETEUIL (1706–1749) A major work by Breteuil was the translation of Newton's "Philosophiae Naturalis Principia Mathematica" into French. Her translation and commentary greatly contributed to the acceptance of Newtonian science in Europe.

See LarsonCalculus.com to read more of this biography.

$$W = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{m} F(c_i) \,\Delta x_i$$
$$= \int_a^b F(x) \,dx.$$

Definition of Work Done by a Variable Force

If an object is moved along a straight line by a continuously varying force F(x), then the **work** *W* done by the force as the object is moved from

$$x = a$$
 to $x = b$
given by
 $W = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} \Delta W$

 $=\int_{a}^{b}F(x) dx.$

The remaining examples in this section use some well-known physical laws. The discoveries of many of these laws occurred during the same period in which calculus was being developed. In fact, during the seventeenth and eighteenth centuries, there was little difference between physicists and mathematicians. One such physicist-mathematician was Emilie de Breteuil. Breteuil was instrumental in synthesizing the work of many other scientists, including Newton, Leibniz, Huygens, Kepler, and Descartes. Her physics text *Institutions* was widely used for many years.

Bettmann/Corbis

is

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1. Hooke's Law: The force F required to compress or stretch a spring (within its elastic limits) is proportional to the distance d that the spring is compressed or stretched from its original length. That is,



where the constant of proportionality k (the spring constant) depends on the specific nature of the spring.

2. Newton's Law of Universal Gravitation: The force F of attraction between two particles of masses m_1 and m_2 is proportional to the product of the masses and inversely proportional to the square of the distance d between the two particles. That is,

$$F = G \frac{m_1 m_2}{d^2}.$$

When m_1 and m_2 are in kilograms and d in meters, F will be in newtons for a value of $G = 6.67 \times 10^{-11}$ cubic meter per kilogram-second squared, where G is the **gravitational constant.**

3. Coulomb's Law: The force F between two charges q_1 and q_2 in a vacuum is proportional to the product of the charges and inversely proportional to the square of the distance d between the two charges. That is,

$$F = k \frac{q_1 q_2}{d^2}.$$

When q_1 and q_2 are given in electrostatic units and d in centimeters, F will be in dynes for a value of k = 1.

EXAMPLE 2 Compressing a Spring

•••• See LarsonCalculus.com for an interactive version of this type of example.

A force of 750 pounds compresses a spring 3 inches from its natural length of 15 inches. Find the work done in compressing the spring an additional 3 inches.

Solution By Hooke's Law, the force F(x) required to compress the spring x units (from its natural length) is F(x) = kx. Because F(3) = 750, it follows that

$$F(3) = (k)(3)$$
 \implies 750 = 3k \implies 250 = k.

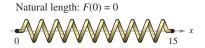
So, F(x) = 250x, as shown in Figure 7.48. To find the increment of work, assume that the force required to compress the spring over a small increment Δx is nearly constant. So, the increment of work is

 $\Delta W = (\text{force})(\text{distance increment}) = (250x) \Delta x.$

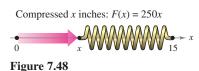
Because the spring is compressed from x = 3 to x = 6 inches less than its natural length, the work required is

$$W = \int_{a}^{b} F(x) \, dx = \int_{3}^{6} 250x \, dx = 125x^{2} \Big]_{3}^{6} = 4500 - 1125 = 3375 \text{ inch-pounds.}$$

Note that you do *not* integrate from x = 0 to x = 6 because you were asked to determine the work done in compressing the spring an *additional* 3 inches (not including the first 3 inches).







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In 2011, China launched an 8.5-ton space module. The module will be used to conduct tests as China prepares to build a space station between 2020 and 2022.

EXAMPLE 3

Moving a Space Module into Orbit

A space module weighs 15 metric tons on the surface of Earth. How much work is done in propelling the module to a height of 800 miles above Earth, as shown in Figure 7.49? (Use 4000 miles as the radius of Earth. Do not consider the effect of air resistance or the weight of the propellant.)

Solution Because the weight of a body varies inversely as the square of its distance from the center of Earth, the force F(x) exerted by gravity is

$$F(x) = \frac{C}{x^2}$$

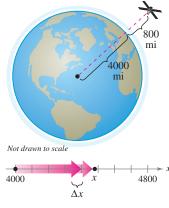


Figure 7.49

work

where C is the constant of proportionality. Because the module weighs 15 metric tons on the surface of Earth and the radius of Earth is approximately 4000 miles, you have

$$15 = \frac{C}{(4000)^2} \implies 240,000,000 = C.$$

So, the increment of work is

$$\Delta W = (\text{force})(\text{distance increment}) = \frac{240,000,000}{x^2} \Delta x.$$

Finally, because the module is propelled from x = 4000 to x = 4800 miles, the total work done is

$$W = \int_{a}^{b} F(x) dx$$
 Formula for

$$= \int_{4000}^{4800} \frac{240,000,000}{x^{2}} dx$$

$$= \frac{-240,000,000}{x} \int_{4000}^{4800}$$
 Integrate.

$$= -50,000 + 60,000$$

$$= 10,000 \text{ mile-tons}$$

$$\approx 1.164 \times 10^{11} \text{ foot-pounds.}$$

In SI units, using a conversion factor of 1 foot-pound ≈ 1.35582 joules, the work done is

$$W \approx 1.578 \times 10^{11}$$
 joules.

The solutions to Examples 2 and 3 conform to our development of work as the summation of increments in the form

 $\Delta W = (\text{force})(\text{distance increment}) = (F)(\Delta x).$

Another way to formulate the increment of work is

 $\Delta W = (\text{force increment})(\text{distance}) = (\Delta F)(x).$

This second interpretation of ΔW is useful in problems involving the movement of nonrigid substances such as fluids and chains.

AFP Creative/Getty Images

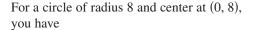
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EXAMPLE 4 Emptying a Tank of Oil

A spherical tank of radius 8 feet is half full of oil that weighs 50 pounds per cubic foot. Find the work required to pump oil out through a hole in the top of the tank.

Solution Consider the oil to be subdivided into disks of thickness Δy and radius *x*, as shown in Figure 7.50. Because the increment of force for each disk is given by its weight, you have

$$\Delta F = \text{weight}$$
$$= \left(\frac{50 \text{ pounds}}{\text{cubic foot}}\right) (\text{volume})$$
$$= 50(\pi x^2 \Delta y) \text{ pounds.}$$



$$x^{2} + (y - 8)^{2} = 8^{2}$$
$$x^{2} = 16y - y^{2}$$

and you can write the force increment as

$$\Delta F = 50(\pi x^2 \Delta y)$$

= 50\pi (16y - y^2) \Delta y.

In Figure 7.50, note that a disk y feet from the bottom of the tank must be moved a distance of (16 - y) feet. So, the increment of work is

$$\Delta W = \Delta F(16 - y)$$

= 50\pi (16y - y²) \Delta y(16 - y)
= 50\pi (256y - 32y² + y³) \Delta y.

Because the tank is half full, *y* ranges from 0 to 8, and the work required to empty the tank is

$$W = \int_0^8 50\pi (256y - 32y^2 + y^3) \, dy$$

= $50\pi \left[128y^2 - \frac{32}{3}y^3 + \frac{y^4}{4} \right]_0^8$
= $50\pi \left(\frac{11,264}{3} \right)$
 $\approx 589,782$ foot-pounds.

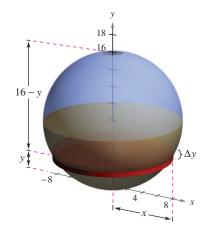
To estimate the reasonableness of the result in Example 4, consider that the weight of the oil in the tank is

$$\left(\frac{1}{2}\right)$$
(volume)(density) = $\frac{1}{2}\left(\frac{4}{3}\pi 8^3\right)$ (50) \approx 53,616.5 pounds

Lifting the entire half-tank of oil 8 feet would involve work of

$$W = FD$$
 Formula for work done by a constant force
 $\approx (53,616.5)(8)$
 $= 428,932$ foot-pounds.

Because the oil is actually lifted between 8 and 16 feet, it seems reasonable that the work done is about 589,782 foot-pounds.



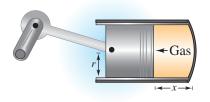






Work required to raise one end of the chain

Figure 7.51



Work done by expanding gas **Figure 7.52**

EXAMPLE 5

Lifting a Chain

A 20-foot chain weighing 5 pounds per foot is lying coiled on the ground. How much work is required to raise one end of the chain to a height of 20 feet so that it is fully extended, as shown in Figure 7.51?

Solution Imagine that the chain is divided into small sections, each of length Δy . Then the weight of each section is the increment of force

$$\Delta F = (\text{weight}) = \left(\frac{5 \text{ pounds}}{\text{foot}}\right)(\text{length}) = 5 \Delta y.$$

Because a typical section (initially on the ground) is raised to a height of *y*, the increment of work is

 $\Delta W = (\text{force increment})(\text{distance}) = (5 \Delta y)y = 5y \Delta y.$

Because y ranges from 0 to 20, the total work is

$$W = \int_0^{20} 5y \, dy = \frac{5y^2}{2} \Big]_0^{20} = \frac{5(400)}{2} = 1000 \text{ foot-pounds.}$$

In the next example, you will consider a piston of radius r in a cylindrical casing, as shown in Figure 7.52. As the gas in the cylinder expands, the piston moves, and work is done. If p represents the pressure of the gas (in pounds per square foot) against the piston head and V represents the volume of the gas (in cubic feet), then the work increment involved in moving the piston Δx feet is

 $\Delta W = (\text{force})(\text{distance increment}) = F(\Delta x) = p(\pi r^2) \Delta x = p \Delta V.$

So, as the volume of the gas expands from V_0 to V_1 , the work done in moving the piston is

$$W = \int_{V_0}^{V_1} p \ dV.$$

Assuming the pressure of the gas to be inversely proportional to its volume, you have p = k/V and the integral for work becomes

$$W = \int_{V_0}^{V_1} \frac{k}{V} dV.$$

EXAMPLE 6

Work Done by an Expanding Gas

A quantity of gas with an initial volume of 1 cubic foot and a pressure of 500 pounds per square foot expands to a volume of 2 cubic feet. Find the work done by the gas. (Assume that the pressure is inversely proportional to the volume.)

Solution Because p = k/V and p = 500 when V = 1, you have k = 500. So, the work is

$$W = \int_{V_0}^{V_1} \frac{k}{V} dV$$
$$= \int_1^2 \frac{500}{V} dV$$
$$= 500 \ln|V| \Big]_1^2$$
$$\approx 246.6 \text{ foot row}$$

 \approx 346.6 foot-pounds.

7.5 Exercises See C

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

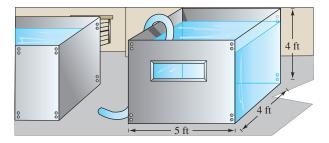
Constant Force In Exercises 1–4, determine the work done by the constant force.

- 1. A 1200-pound steel beam is lifted 40 feet.
- 2. An electric hoist lifts a 2500-pound car 6 feet.
- **3.** A force of 112 newtons is required to slide a cement block 8 meters in a construction project.
- **4.** The locomotive of a freight train pulls its cars with a constant force of 9 tons a distance of one-half mile.

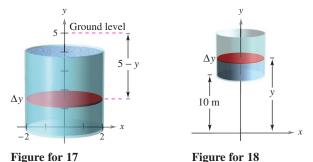
Hooke's Law In Exercises 5–10, use Hooke's Law to determine the variable force in the spring problem.

- **5.** A force of 5 pounds compresses a 15-inch spring a total of 3 inches. How much work is done in compressing the spring 7 inches?
- **6.** A force of 250 newtons stretches a spring 30 centimeters. How much work is done in stretching the spring from 20 centimeters to 50 centimeters?
- **7.** A force of 20 pounds stretches a spring 9 inches in an exercise machine. Find the work done in stretching the spring 1 foot from its natural position.
- **8.** An overhead garage door has two springs, one on each side of the door. A force of 15 pounds is required to stretch each spring 1 foot. Because of the pulley system, the springs stretch only one-half the distance the door travels. The door moves a total of 8 feet, and the springs are at their natural length when the door is open. Find the work done by the pair of springs.
- **9.** Eighteen foot-pounds of work is required to stretch a spring 4 inches from its natural length. Find the work required to stretch the spring an additional 3 inches.
- **10.** Seven and one-half foot-pounds of work is required to compress a spring 2 inches from its natural length. Find the work required to compress the spring an additional one-half inch.
- **11. Propulsion** Neglecting air resistance and the weight of the propellant, determine the work done in propelling a five-ton satellite to a height of (a) 100 miles above Earth and (b) 300 miles above Earth.
- **12. Propulsion** Use the information in Exercise 11 to write the work *W* of the propulsion system as a function of the height *h* of the satellite above Earth. Find the limit (if it exists) of *W* as *h* approaches infinity.
- **13. Propulsion** Neglecting air resistance and the weight of the propellant, determine the work done in propelling a 10-ton satellite to a height of (a) 11,000 miles above Earth and (b) 22,000 miles above Earth.
- **14. Propulsion** A lunar module weighs 12 tons on the surface of Earth. How much work is done in propelling the module from the surface of the moon to a height of 50 miles? Consider the radius of the moon to be 1100 miles and its force of gravity to be one-sixth that of Earth.

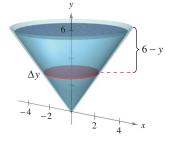
15. Pumping Water A rectangular tank with a base 4 feet by 5 feet and a height of 4 feet is full of water (see figure). The water weighs 62.4 pounds per cubic foot. How much work is done in pumping water out over the top edge in order to empty (a) half of the tank and (b) all of the tank?



- **16. Think About It** Explain why the answer in part (b) of Exercise 15 is not twice the answer in part (a).
- **17. Pumping Water** A cylindrical water tank 4 meters high with a radius of 2 meters is buried so that the top of the tank is 1 meter below ground level (see figure). How much work is done in pumping a full tank of water up to ground level? (The water weighs 9800 newtons per cubic meter.)



- **18. Pumping Water** Suppose the tank in Exercise 17 is located on a tower so that the bottom of the tank is 10 meters above the level of a stream (see figure). How much work is done in filling the tank half full of water through a hole in the bottom, using water from the stream?
- **19. Pumping Water** An open tank has the shape of a right circular cone (see figure). The tank is 8 feet across the top and 6 feet high. How much work is done in emptying the tank by pumping the water over the top edge?



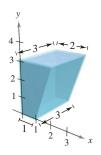
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20. Pumping Water Water is pumped in through the bottom of the tank in Exercise 19. How much work is done to fill the tank

(a) to a depth of 2 feet?

(b) from a depth of 4 feet to a depth of 6 feet?

- **21. Pumping Water** A hemispherical tank of radius 6 feet is positioned so that its base is circular. How much work is required to fill the tank with water through a hole in the base when the water source is at the base?
- **22. Pumping Diesel Fuel** The fuel tank on a truck has trapezoidal cross sections with the dimensions (in feet) shown in the figure. Assume that the engine is approximately 3 feet above the top of the fuel tank and that diesel fuel weighs approximately 53.1 pounds per cubic foot. Find the work done by the fuel pump in raising a full tank of fuel to the level of the engine.



Pumping Gasoline In Exercises 23 and 24, find the work done in pumping gasoline that weighs 42 pounds per cubic foot. (*Hint:* Evaluate one integral by a geometric formula and the other by observing that the integrand is an odd function.)

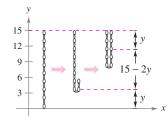
- **23.** A cylindrical gasoline tank 3 feet in diameter and 4 feet long is carried on the back of a truck and is used to fuel tractors. The axis of the tank is horizontal. The opening on the tractor tank is 5 feet above the top of the tank in the truck. Find the work done in pumping the entire contents of the fuel tank into the tractor.
- **24.** The top of a cylindrical gasoline storage tank at a service station is 4 feet below ground level. The axis of the tank is horizontal and its diameter and length are 5 feet and 12 feet, respectively. Find the work done in pumping the entire contents of the full tank to a height of 3 feet above ground level.

Lifting a Chain In Exercises 25–28, consider a 20-foot chain that weighs 3 pounds per foot hanging from a winch 20 feet above ground level. Find the work done by the winch in winding up the specified amount of chain.

- 25. Wind up the entire chain.
- 26. Wind up one-third of the chain.
- **27.** Run the winch until the bottom of the chain is at the 10-foot level.
- 28. Wind up the entire chain with a 500-pound load attached to it.

Lifting a Chain In Exercises 29 and 30, consider a 15-foot hanging chain that weighs 3 pounds per foot. Find the work done in lifting the chain vertically to the indicated position.

29. Take the bottom of the chain and raise it to the 15-foot level, leaving the chain doubled and still hanging vertically (see figure).

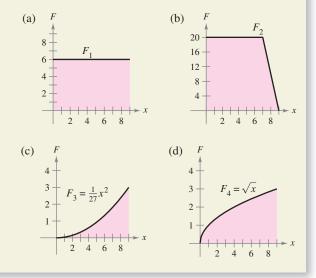


30. Repeat Exercise 29 raising the bottom of the chain to the 12-foot level.

WRITING ABOUT CONCEPTS

- **31. Work by a Constant Force** State the definition of work done by a constant force.
- **32. Work by a Variable Force** State the definition of work done by a variable force.
- **33. Work** Which of the following requires more work? Explain your reasoning.
 - (a) A 60-pound box of books is lifted 3 feet.
 - (b) A 60-pound box of books is held 3 feet in the air for 2 minutes.

HOW DO YOU SEE IT? The graphs show the force F_i (in pounds) required to move an object 9 feet along the *x*-axis. Order the force functions from the one that yields the least work to the one that yields the most work without doing any calculations. Explain your reasoning.



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- **35. Ordering Forces** Verify your answer to Exercise 34 by calculating the work for each force function.
- **36.** Electric Force Two electrons repel each other with a force that varies inversely as the square of the distance between them. One electron is fixed at the point (2, 4). Find the work done in moving the second electron from (-2, 4) to (1, 4).

Boyle's Law In Exercises 37 and 38, find the work done by the gas for the given volume and pressure. Assume that the pressure is inversely proportional to the volume. (See Example 6.)

- **37.** A quantity of gas with an initial volume of 2 cubic feet and a pressure of 1000 pounds per square foot expands to a volume of 3 cubic feet.
- **38.** A quantity of gas with an initial volume of 1 cubic foot and a pressure of 2500 pounds per square foot expands to a volume of 3 cubic feet.
- **Hydraulic Press** In Exercises 39-42, use the integration capabilities of a graphing utility to approximate the work done by a press in a manufacturing process. A model for the variable force *F* (in pounds) and the distance *x* (in feet) the press moves is given.

Force	Interval
39. $F(x) = 1000[1.8 - \ln(x + 1)]$	$0 \le x \le 5$
40. $F(x) = \frac{e^{x^2} - 1}{100}$	$0 \le x \le 4$
41. $F(x) = 100x\sqrt{125 - x^3}$	$0 \le x \le 5$
42. $F(x) = 1000 \sinh x$	$0 \le x \le 2$

- **43.** Modeling Data The hydraulic cylinder on a woodsplitter has a 4-inch bore (diameter) and a stroke of 2 feet. The hydraulic pump creates a maximum pressure of 2000 pounds per square inch. Therefore, the maximum force created by the cylinder is $2000(\pi 2^2) = 8000\pi$ pounds.
 - (a) Find the work done through one extension of the cylinder, given that the maximum force is required.
 - (b) The force exerted in splitting a piece of wood is variable. Measurements of the force obtained in splitting a piece of wood are shown in the table. The variable *x* measures the extension of the cylinder in feet, and *F* is the force in pounds. Use Simpson's Rule to approximate the work done in splitting the piece of wood.

x	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
F(x)	0	20,000	22,000	15,000	10,000	5000	0

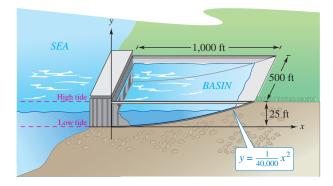
- (c) Use the regression capabilities of a graphing utility to find a fourth-degree polynomial model for the data. Plot the data and graph the model.
- (d) Use the model in part (c) to approximate the extension of the cylinder when the force is maximum.
- (e) Use the model in part (c) to approximate the work done in splitting the piece of wood.

Andrew J. Martinez/Photo Researchers, Inc

SECTION PROJECT

Tidal Energy

Tidal power plants use "tidal energy" to produce electrical energy. To construct a tidal power plant, a dam is built to separate a basin from the sea. Electrical energy is produced as the water flows back and forth between the basin and the sea. The amount of "natural energy" produced depends on the volume of the basin and the tidal range—the vertical distance between high and low tides. (Several natural basins have tidal ranges in excess of 15 feet; the Bay of Fundy in Nova Scotia has a tidal range of 53 feet.)



- (a) Consider a basin with a rectangular base, as shown in the figure. The basin has a tidal range of 25 feet, with low tide corresponding to y = 0. How much water does the basin hold at high tide?
- (b) The amount of energy produced during the filling (or the emptying) of the basin is proportional to the amount of work required to fill (or empty) the basin. How much work is required to fill the basin with seawater? (Use a seawater density of 64 pounds per cubic foot.)



The Bay of Fundy in Nova Scotia has an extreme tidal range, as displayed in the greatly contrasting photos above.

FOR FURTHER INFORMATION For more information on tidal power, see the article "LaRance: Six Years of Operating a Tidal Power Plant in France" by J. Cotillon in *Water Power Magazine*.